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ON POSSIBLE MANIFESTATION OF FEEDBACK COUPLING BETWEEN GEOMETRY AND MATTER IN A PHENOMENON OF AN ACCELERATING EXPANSION OF THE UNIVERSE

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Abstract

It is shown that an accelerating expansion of the present-day Universe extracted from observed luminosity of the type Ia supernovae can be explained by quantum theory which takes into account feedback coupling between geometry and matter (like in Mach's principle). At the same time an accelerating expansion of the Universe is explained by the influence of small negative cosmological constant. A comparison with the model with positive cosmological constant (dark energy) which also has obtained its theoretical grounds in a structure of the developed formalism is made. Parameters of the Universe in the states with large quantum numbers are calculated.

1. INTRODUCTION

An analysis of possible reasons for the observed weak luminosities of the type Ia supernovae (SNe Ia) at cosmological redshift $z \approx 0.5$ [1, 2] demonstrates that this phenomenon cannot be put down to nonstandard evolution of their luminosity, absorption effects of an interstellar dust, gravitational lensing and other physical processes which are not connected with the overall expansion of the Universe as a whole (see discussion in [3, 4, 5]). In accordance with the principles of general relativity the observed dimming of the SNe Ia can be interpreted as an evidence of an accelerating expansion of the present-day Universe. Phenomenological models which are used herewith suppose an existence of nonzero cosmological constant in the Universe treated as vacuum energy density or hypothetical cosmological liquid with negative pressure (so-called dark energy [6, 7]). The models of such type allow to describe the available dataset on the distance moduli of the SNe Ia depending on redshift by fixing free parameters (e.g. from a χ^2 statistic).

Providing a formal agreement with modern astrophysical observational data (SCP [2], HST [4] and WMAP [8] projects) phenomenological models come across difficulties in questions of principle when trying to find a theoretical explanation for the values of their own free parameters and their physical motivation. Among the fundamental problems available here it is possible to pick out the cosmological constant problem, a task to determine the nature of dark energy and a puzzle concerning the coincidence between the contributions from dark energy $\Omega_X \approx 0.7$ and dark matter $\Omega_M \approx 0.3$ to the total energy density nowadays [4, 5, 9, 10]. It is assumed that in order to solve them one should exceed the limits of modern cosmology built on the principles of general relativity [10, 11, 12].

In the present article the problem of an accelerating expansion of the Universe is analysed within the framework of cosmological quantum model [13, 14, 15, 16]. The main feature of this approach lies in taking into account possible feedback coupling between geometry and matter. This coupling should be taken into consideration when one studies the processes in which the Universe appears as a whole (on the scales that exceed significantly the size of the superclusters of galaxies, > 200 Mpc).

Quantum model of the Universe characterized by nonzero vacuum energy density, $\rho_{vac} \neq 0$, which takes into account feedback coupling between geometry and matter, allows to describe numerically the observed dependence of the distance moduli of the SNe Ia on z in the whole range of redshift measured values [5] with the same accuracy that is achieved within the limits of the phenomenological model with positive cosmological constant in classical cosmology. The latter also receives its theoretical grounds in the structure of developed formalism.

2. Quantum geometrodynamics in the minisuperspace model

2.1 Motivation

An available current experimental dataset allows to state that quantum theory describes adequately properties of various physical systems. The universal validity of quantum theory demands that the Universe as a whole must obey quantum laws too. The quest for these laws falls into the realm of research of quantum cosmology. Since gravity dominates on cosmological (very large) scales any consistent formalism of quantum cosmology must contain quantum theory of gravity. Driving forces that give reasons for quantum gravity research are not restricted to aspirations to obtain a unified theory

of all interactions, search for mathematical consistency, or determination of origin and nature of space and time (a review of motivations from different points of view on the problem one can find, e.g., in [17]). There exist the problems which remain unsolved in standard model of the hot Universe and which, as it seems today, cannot be solved without appeal to quantum cosmology.

It is generally accepted that the early stage of an exponential expansion of the Universe within the framework of inflationary scenario of classical cosmology withdraws the horizon problem, directs the density parameter Ω to unity and explains the absence of registration acts of monopoles, topological defects and etc. by very low density of these relics. But a number of problems remains outside the limits of inflationary model. Among them there are the mystery of origin of primordial fluctuations of energy density, explanation of time arrow and determination of initial conditions of the evolution of the Universe [18]. Besides this, it is established nowadays that certain problems are solved by inflationary model in improper way or these problems can be avoided at all or solved differently [16, 19]. For instance, the horizon problem is in fact directly connected with the physical processes in the Planck era [20] and therefore one should appeal to quantum gravity in order to solve it. The flatness problem can obtain its solution within quantum description in Planck time as well [21].

Inflationary scenario does not allow to tackle the problem of presence of singularities in quantum cosmology [19]. Inflation cannot be continued infinitely into the past, mainly because the flat de Sitter metric becomes geodesically incomplete then [22]. The main achievement of inflationary model is, as it is widely accepted, an opportunity to obtain the Universe with current parameters (such as size, energy density contrast, age etc.) starting from the natural Planck values for different quantities (so-called *small bang*). However, the same result can be achieved in quantum cosmology as well [13, 14], which does not contradict with inflationary paradigm at this point. Let us note that inflationary model itself needs quantum cosmology for its motivation, namely in order to ensure the long enough duration of the inflation period (determined by numerical coefficient in exponent) which would agree with observations [18].

Since cosmology considers the Universe as a system with very large scales, while quantum phenomena are typical for microscopic systems, then a combination of words *quantum* and *cosmology* may seem contradictory. It is commonly accepted that quantum effects of gravity take place at Planck length scales $L_P = \sqrt{G\hbar/c^3} \sim 10^{-33}$ cm, while the space-time structure at large scales

will be classical automatically. Such point of view is motivated only if a consistent quantum theory of gravity exists within whose framework the small parameter L_P/L , where L is some typical length in the Universe, can be introduced and an appropriate perturbation theory can be constructed, and from the latter it follows that quantum effects are negligibly small for $L_P/L \ll 1$ [17]. But an assumption that quantum effects of gravity are small neglects completely the possibility of their non-perturbative character, while if one takes it into account that could provide in particular quantum field theory with an appropriate cut-off.

But it should be noticed that quantum effects are not a priori restricted to certain scales. Rather the processes of decoherence (when the coherent superposition of the states turns into incoherent one; interference effects are absent) through the environment can explain why quantum effects are negligible or important for an object under consideration [23, 24, 25].

Since nowadays a consistent quantum theory of gravity has not been formulated, a research here is conducted in a few directions. The method of canonical quantization of constraint systems proposed by Dirac [26] provides a basis for the theory developed in this article.

The structure of constraints which describe the evolution of intrinsic geometry and extrinsic curvature of spacelike hypersurface in space-time is such that true dynamical degrees of freedom cannot be distinguished explicitly from quantities which determine hypersurface. This leads to famous problems in interpretation of quantum geometrodynamics constructed on the basis of the Wheeler–DeWitt equation [27]. The main reason for these difficulties is that there is no predetermined way to identify spacetime events in generally covariant theories (i.e. one cannot measure the metric, but only the geometry).

In order to solve the problem mentioned above an approach related to the notion of a medium which determines the reference frame¹ (so-called *reference fluid*) seems promising [30, 31, 32, 33]. The problem here lies in finding an appropriate medium (an additional source in the Einstein equations) which, when quantizing in Dirac’s formalism, would lead to functional Schrödinger type equation. Variables which describe a medium (the reference frame is considered as a dynamical system) mark spacetime events. They play the role of the canonical coordinates which determine *embedding* in surround-

¹Application of *material reference frames* has a long history. They were used already by Einstein [28] and Hilbert [29], but in somewhat idealized form which did not take into account a back action of material reference frames on geometry.

ing spacetime, while new constraints turn out to be linear with respect to momenta canonically conjugate with the medium variables. Such an additional source is introduced in the action and determines in particular the time variable. The invariance of action here remains unbroken.

The replacement of the Wheeler–DeWitt equation by the functional Schrödinger type equation allows to introduce the positive-definite conserved inner product and to advance essentially in constructing of consistent quantum theory of gravity. But a discovery of corresponding (physical) medium, which defines the reference frame, is a nontrivial task in itself. In Refs. [13, 14, 15, 16] this problem was solved in terms of the minisuperspace model. We shall consider this case in more detail.

2.2 Main equations

Just the same as in ordinary nonrelativistic and relativistic theories it is possible to assume that the problem of evolution and research into the properties of the Universe as a whole can be reduced to solution of a partial differential equation which determines eigenvalues and eigenfunctions of some Hamiltonian-like operator (in the space of generalized variables whose roles are played by metric tensor components and matter fields). For simplicity we restrict our study to the case of minimal coupling between geometry and matter. Taking into account that scalar fields play fundamental roles both in quantum field theory (see, e.g., [34]) and in cosmology of the early Universe [35, 36, 37], we assume that the Universe is filled *ab initio* by primordial matter in the form of a scalar field ϕ with some potential energy density $V(\phi)$.

We suppose that the Universe as a whole is homogeneous, isotropic and spatially flat, and a scalar field ϕ is uniform. The geometry of such a Universe is determined by the famous Robertson–Walker metric [38]. From the principle of least action it follows the constraint equation [13, 16, 39], $\delta S/\delta N = 0$, where S is a corresponding action functional, and N is a lapse function that specifies the time reference scale and plays the role of a Lagrange multiplier in the ADM formalism [40], which is the Einstein–Friedmann equation for the $(^0_0)$ component.

The structure of the constraint is such that true dynamical degrees of freedom cannot be singled out explicitly. In the model considered, this difficulty is reflected in that the choice of the time variable is ambiguous (so-called *problem of time*). In order to solve this problem in general relativity it will be enough to supplement the field equations with a coordinate condition which does

not change the Einstein equations themselves, but only specifies the spacetime platform from which one observes the gravitational field (the enlarged system of constraints is no longer first class and it is possible to eliminate *non-dynamical* variables). But this method does not allow to solve the problem of time for a quantum description [30].

Therefore we shall use another approach in which a coordinate condition is imposed prior to varying the action functional and included in it with the aid of a Lagrange multiplier. Parametrization of the action functional (see, e.g., [26, 30, 41]) restores its coordinate invariance expressing it in arbitrary coordinates. At the same time the *privileged* time coordinate introduced by means of the coordinate condition is adjoined to the field variables and takes the role of the medium variable which determines the reference frame.

We will choose the coordinate condition in the form

$$T' = N, \quad (1)$$

where T is the new field variable (the privileged time coordinate), while differentiation with respect to arbitrary variable (conformal time or *arc parameter*) η is denoted by a prime, the parameter η is related to the synchronous proper time t by the differential equation $dt = N a d\eta$, a is a cosmological scale factor.

We shall include the coordinate condition (1) in the action functional with the aid of a Lagrange multiplier P and obtain the modified action of the minisuperspace model in the conventional form

$$S_{mod} = \int d\eta [\pi_a a' + \pi_\phi \phi' + P T' - N H], \quad (2)$$

where π_a and π_ϕ are the momenta canonically conjugate with the variables a and ϕ , and

$$H = \frac{1}{2} \left(-\pi_a^2 + \frac{2}{a^2} \pi_\phi^2 - a^2 + a^4 V(\phi) \right) + P \quad (3)$$

is the Hamiltonian to within multiplier N . Here and below we give all relations between dimensionless units. The length is taken in units of the modified Planck length $l_P = \sqrt{2G\hbar/(3\pi c^3)} = 0.744 \times 10^{-33}$ cm, the energy density is measured in units of $\rho_P = 3c^4/(8\pi G l_P^2) = 1.627 \times 10^{117}$ GeV cm⁻³ and so on.

The variation of the action (2) with respect to N leads to the constraint equation

$$\delta S_{mod}/\delta N = 0 \quad \Rightarrow \quad H = 0. \quad (4)$$

The parameter T can be used as an independent variable for the description of the evolution of the Universe both in classical and quantum cosmology [16].

In quantum theory the constraint equation (4), in accordance with a procedure proposed by Dirac [26], comes to be a constraint on the wavefunction Ψ ,

$$i \partial_T \Psi = \hat{\mathcal{H}} \Psi, \quad (5)$$

with a Hamiltonian-like operator

$$\hat{\mathcal{H}} = \frac{1}{2} \left(\partial_a^2 - \frac{2}{a^2} \partial_\phi^2 - a^2 + a^4 V(\phi) \right), \quad (6)$$

where we have introduced the operators $P = -i \partial_T$, $\pi_a = -i \partial_a$ and $\pi_\phi = -i \partial_\phi$, which satisfy the ordinary canonical commutation relations, $[T, P] = i$, $[a, \pi_a] = i$, $[\phi, \pi_\phi] = i$, while others vanish.

The wavefunction Ψ depends on a cosmological scale factor a , a scalar field ϕ and time coordinate T . One can introduce, at least formally, a positive definite scalar product $\langle \Psi | \Psi \rangle < \infty$ and specify the norm of a state. This makes it possible to define a Hilbert space of physical states and to construct quantum mechanics for the model of the Universe being considered.

Equation (5) has a particular solution with separable variables

$$\Psi = e^{\frac{i}{2} E T} \psi_E, \quad (7)$$

where the wavefunction ψ_E satisfies the time-independent equation

$$\left(-\partial_a^2 + \frac{2}{a^2} \partial_\phi^2 + U - E \right) \psi_E = 0, \quad (8)$$

and

$$U = a^2 - a^4 V(\phi) \quad (9)$$

can be interpreted as an effective potential.

The function ψ_E is specified in space of two variables, a and ϕ . In classical approximation the eigenvalue E determines the components of the energy-momentum tensor

$$\begin{aligned} \tilde{T}_0^0 &= \frac{E}{a^4}, & \tilde{T}_1^1 &= \tilde{T}_2^2 = \tilde{T}_3^3 = -\frac{E}{3a^4}, \\ \tilde{T}_\nu^\mu &= 0 \quad \text{for } \mu \neq \nu, \end{aligned} \quad (10)$$

which in the case $E > 0$ describes an additional source of the gravitational field in the form of relativistic matter of an arbitrary nature. Equation (8) formally turns into the Wheeler–DeWitt equation for the minisuperspace model [27] in the special case $E \rightarrow 0$.

2.3 Model of a scalar field

The quantum state ψ_E depends on the form and numerical value of $V(\phi)$. We shall use the model of a scalar field which slowly (in comparison with rapid motion with respect to the variable a) rolls from some initial value ϕ_{start} with the Planck energy density $V(\phi_{start}) \sim 1^2$ to the equilibrium state ϕ_{vac} with the energy density $\rho_{vac} = V(\phi_{vac}) \ll 1$. This constant density determines a cosmological constant $\Lambda = 3\rho_{vac}$. At the next stage of the evolution the scalar field oscillates with a small amplitude near ϕ_{vac} under the action of quantum fluctuations. In such a model the motion with respect to ϕ always will be finite.

The analogous model of the scalar field was considered for the first time in connection with inflationary scenario (see, e.g., [36, 37] and references therein)³. For inflationary model the presence of minimum in the function $V(\phi)$ is of great importance. The oscillations of the scalar field near a state of equilibrium with subsequent transfer of energy of these oscillations to *real* particles allow to fill the Universe, which has become empty after the exponential expansion, with hot matter [36, 43].

2.4 Solution of the time-independent equation

For positive definite function $V(\phi)$ an effective potential U as a function of a has a form of barrier. In this case the Universe described by equation (8) can be both in continuum states with $E > 0$ and quasistationary ones which correspond to complex values $E = E_n + i \Gamma_n$, where $E_n > 0$, $\Gamma_n > 0$ and $\Gamma_n \ll E_n$ [14, 16]. Quasistationary states are the most interesting since the Universe in such states can be described by the set of standard cosmological parameters (Hubble constant, deceleration parameter, mean energy density, density contrast, amplitude of fluctuations of the cosmic microwave background radiation temperature and so on) [16].

Taking into account that a motion with respect to a in the early Universe is supposed to be rapid in comparison

²The evolution of the Universe in time can be considered in accordance with classical conceptions starting from this value of the energy density [36].

³We shall note that in light of the coincidence problem (the contributions from dark energy and dark matter to the total energy density in the Universe have the same order of magnitude), the model of *quintessence* - it is a scalar field φ of special type with potential energy density $\mathcal{V}(\varphi)$ modeled by different functions (see, e.g., review [9]) - is widely discussed in the literature. There is a fundamental difference between the quintessence φ and the scalar field ϕ of the model being considered in this article. The field ϕ models primordial matter which is a source of real matter [42], including the quintessence φ (if it really exists). The question concerning the relation between the fields ϕ and φ goes beyond the scope of this article and will not be considered further.

with the slow variation of the state of the scalar field we find that the wavefunction ψ_E of quasistationary state, considered as a function of a at fixed field ϕ , has a sharp peak and it is concentrated mainly in the region limited by the barrier (9) [16, 39]. Then following Fock [44] one can introduce an approximate function which is equal to exact wavefunction inside the barrier and vanishes outside it. Taking into account finite motion with respect to ϕ , this function can be normalized and used in calculations of expectation values of observed parameters. Such an approximation does not take into account exponentially small probability of tunneling through the barrier U in the region of large values of a , where $a^2 V > 1$ [14, 16]. It is valid for calculations of mean observed parameters of the Universe within its lifetime in given quasistationary state⁴ when this state can be considered as stationary one. Here we have a close analogy with the corresponding conclusions of ordinary quantum mechanics [45]. In the region of large values of a outside the barrier the WKB approximation is valid [13] and the solution of equation (8) can be written in an explicit form [13, 14, 16, 39].

Let us consider the solution of equation (8) in the approximation of finite motion with respect to variables a and ϕ . It is convenient to expand the wavefunction ψ_E on the basis of the functions $\langle a|n\rangle$ of oscillator

$$(-\partial_a^2 + a^2 - \epsilon_n^0) |n\rangle = 0, \quad (11)$$

where $a \geq 0$, $\epsilon_n^0 = 4n + 3$, $n = 0, 1, 2, \dots$ is a number of state. This expansion has the form

$$\psi_E = \sum_n |n\rangle f_n. \quad (12)$$

Functions $f_n(\phi)$ satisfy the set of differential equations

$$\partial_\phi^2 f_n + \frac{1}{2} \sum_n K_{nn'} f_{n'} = 0 \quad (13)$$

with the kernel

$$K_{nn'}(\phi; E) = \langle n|a^2|n'\rangle [\epsilon_n^0 - E] - \langle n|a^6|n'\rangle V(\phi). \quad (14)$$

In classical theory the gravitational field is determined by the spacetime metric [46]. According to (11) the states $\langle a|n\rangle$ will describe geometrical properties of the Universe as a whole in quantum theory. A motion with respect to a can be quantized. The correspondent equidistant spectrum of energy has the form $\mathcal{E}_n =$

$m_P(N + \frac{1}{2})$, where m_P is the Planck mass, and $N = 2n + 1$ gives the number of elementary quantum excitations of the vibrations of oscillator (11). Mass of elementary excitations of geometry coincides with mass of known Markov maximons which are particle-like formations with the Planck mass⁵.

3. Universe in the states with large quantum numbers

Bearing in mind future application of the developed formalism to the interpretation of astrophysical observational data for our Universe we shall consider the cosmological equations obtained above in the approximation of large quantum numbers.

Direct calculations [13, 14] demonstrate that in the quantum model of the Universe with the slow-roll potential energy density $V(\phi)$ the first quasistationary state emerges when the density reaches the value $V = 0.08$. This state is characterized by finite values of energy density, $\rho \sim \rho_P$, and scalar curvature, $R \sim l_P^{-2}$, while the singular state with $\rho \sim \infty$ and $R \sim \infty$ is excluded from consideration as non-physical.

When $V(\phi)$ decreases to the value $V \ll 0.1$, the number of available states of the Universe increases up to $n \gg 1$. Before the instant when the scalar field reaches its equilibrium state ϕ_{vac} the Universe may get into the state with the number $n \gg 1$. Really, the origin of new quantum levels and the (exponential) reduction of the width of the states that have emerged earlier lead to a competition between the processes of tunneling through the potential barrier U from a given n -th state and allowed transitions between the states, $n \rightarrow n \pm i$, where $i = 1, 2$ [14, 16]. A comparison between the probabilities of these processes demonstrates that the process $n \rightarrow n + 1$ appears to be the most probable. Such transitions are realized at the expense of energy of the scalar field accumulated in the state ϕ_{start} .

Taking into account an explicit form of the matrix elements $\langle n'|a^2|n\rangle$ and $\langle n'|a^6|n\rangle$, we find that in the limiting case $n \gg 1$ the set of equations (13) is reduced to one equation in the approximation $f_n \approx f_{n \pm j}$, where $j = 1, 2, 3$. This approximation preserves the orthogonality of the states with respect to quantum number (s) that characterizes the field ϕ .

Equation for f_n as a function of new variable $x = \sqrt{m/2} (2N)^{3/4} (\phi - \phi_{vac})$ which describes the deviation

⁴At $V \lesssim 10^{-122}$ this time can reach the values close to the age of our Universe [16].

⁵Let us remind that the notion about massless gravitons as gravitational field quanta was introduced within the framework of the theory constructed in weak gravitational field approximation (gravitational waves). It is obvious that for cosmologically significant effects this approximation is not valid.

of the field ϕ from the equilibrium value ϕ_{vac} has the form

$$[\partial_x^2 + z - v(x)] f_n(x) = 0, \quad (15)$$

where we denote $z = (\sqrt{2N}/m)(1 - E/(2N))$, $v(x) = (2N)^{3/2} V(\phi)/m$, and m is some parameter. It is convenient to choose $m^2 = [\partial_\phi^2 V(\phi)]_{\phi_{vac}} > 0$. We shall assume that the density $V(\phi)$ near the point ϕ_{vac} is a smooth enough function. Then expanding $v(x)$ into Taylor's series near the point $x = 0$, we obtain

$$v(x) = v(0) + x^2 + \alpha x^3 + \beta x^4 + \dots, \quad (16)$$

where

$$\alpha = \frac{\sqrt{2}}{3} m^{-5/2} (2N)^{-3/4} [\partial_\phi^3 V(\phi)]_{\phi_{vac}},$$

$$\beta = \frac{1}{6} m^{-3} (2N)^{-3/2} [\partial_\phi^4 V(\phi)]_{\phi_{vac}}.$$

Since $N \sim n \gg 1$, then $|\alpha| \ll 1$ and $|\beta| \ll 1$, and equation (15) with the potential (16) can be solved using the perturbation theory for stationary problems with a discrete spectrum. We take for the state of the unperturbed problem the state of the harmonic oscillator described by equation (15) with the potential (16) for $\alpha = \beta = 0$. As a result we obtain

$$z = 2s + 1 + v(0) + \Delta z, \quad (17)$$

where $s = 0, 1, 2, \dots$ is number of the states of the field ϕ , Δz takes into account its self-action (an explicit form of Δz is given in [42]). The spectrum of energy states of the field ϕ has the form $M' = M + \Delta M$, where

$$M = m \left(s + \frac{1}{2} \right), \quad (18)$$

and $\Delta M = m\Delta z/2$. One can demonstrate that the following estimation is valid

$$\frac{\Delta M}{M} \ll 1 \quad \text{at} \quad s > m^{-2}. \quad (19)$$

Hence it appears that at large enough values of s one can neglect the self-action of the field ϕ . It is reasonable to interpret M (18) as a quantity of matter/energy in the Universe represented in the form of a sum of elementary quantum excitations of the vibrations of the field ϕ near the equilibrium state ϕ_{vac} with the masses m ; s is the number of such excitations. For instance, for $m \sim 1$ GeV the condition (19) is satisfied at $s > 10^{38}$. Assuming $s \sim$

10^{80} (the equivalent number of baryons in our Universe) we obtain a restriction on mass of quantum excitations from below, $m > 10^{-21}$ GeV.

Taking into account the relation between z and E , from (17) we obtain the expression for the eigenvalue

$$E = 2N - (2N)^2 \rho_{vac} - 2\sqrt{2N} M'. \quad (20)$$

The wavefunction of the Universe in the state with large quantum numbers, $n \gg 1$, $s \gg 1$, has the form

$$\psi_E(a, \phi) = \varphi_n(a) f_{ns}(\phi), \quad (21)$$

where

$$\varphi_n(a) = \left(\frac{4}{2N+1} \right)^{1/4} \cos \left(\sqrt{2N+1} a - \frac{\pi N}{2} \right), \quad (22)$$

$$f_{ns}(\phi) = \left(\frac{m(2N)^{3/2}}{2(2s+1)} \right)^{1/4} \times \cos \left(\sqrt{(2s+1) \frac{m}{2} (2N)^{3/2} (\phi - \phi_{vac})} - \frac{\pi s}{2} \right). \quad (23)$$

These functions are normalized to unity in the ranges $0 \leq a \leq a_c$ and $\phi_- \leq \phi \leq \phi_+$ limited by the classical turning points

$$a_c = \sqrt{2N+1}, \quad \phi_{\pm} = \phi_{vac} \pm \left(\frac{2(2s+1)}{m(2N)^{3/2}} \right)^{1/2}$$

for corresponding oscillator potentials. Beyond these ranges an exact wavefunction decreases exponentially. Here a perfect analogy with the normalization of quasiclassical functions in quantum mechanics may be observed (see, e.g., [47]).

Taking into account that the mean value of the scale factor $\langle a \rangle$ in the state (21) is equal to

$$\langle a \rangle = \frac{1}{2} \sqrt{2N+1}, \quad (24)$$

we come to a conclusion that $v(x)$ in equation (15) is the potential energy of the scalar field contained in the Universe with the volume $\sim \langle a \rangle^3$, and the variable x^2 characterizes the deviation squared of the field ϕ from an equilibrium state in such a volume. Thus equation (15) describes the stationary states which characterize the scalar field ϕ in the Universe as a whole. The quantities $v(x)$, x^2 and M are its overall characteristics.

Taking (24) into account, the condition (20) can be rewritten in the form of feedback coupling relation between geometrical and energetic characteristics of the Universe

$$\langle a \rangle = M + \frac{E}{4\langle a \rangle} + 4\langle a \rangle^3 \rho_{vac}, \quad (25)$$

where we discard a small addition ΔM and take into consideration that $N \gg 1$. Here the second summand on the right describes the energy of relativistic matter, while the third term gives the contribution from the vacuum of the scalar field.

Equation (25) can be interpreted as one of possible implementations of famous Mach's principle [48]. Indeed, passing to dimensional quantities we obtain

$$\frac{G}{c^2} \frac{\mathcal{M}}{\mathcal{R}} \sim 1,$$

where \mathcal{M} and \mathcal{R} are measures of mass (without taking gravitational interaction between bodies into account) and radius of the observed part of the Universe. This relation follows from the Lense-Thirring effect in general relativity as well. In this connection the Universe appears like a huge system which tracks and adjusts its parameters according to feedback coupling condition (25) (see also [48]).

4. Cosmological models

Using the relation for mean values of a product of operators [49]

$$\left\langle -\frac{1}{a^4} \partial_a^2 \right\rangle = \left\langle \left(\frac{1}{a} \frac{da}{dt} \right)^2 \right\rangle,$$

where t is the synchronous proper time, while averaging is performed over the state ψ_E normalized in a way indicated above, from equation (8) one can pass to the relation between expectation values. Assuming that the mean $\langle a \rangle$ in such a state determines the scale factor in classical description in general relativity, we obtain the Einstein-Friedmann equation in terms of mean values

$$\left(\frac{1}{\langle a \rangle} \frac{d\langle a \rangle}{dt} \right)^2 = \langle \rho \rangle - \frac{1}{\langle a \rangle^2}, \quad (26)$$

where

$$\langle \rho \rangle = \frac{2}{\langle a \rangle^6} \langle -\partial_\phi^2 \rangle + \langle V \rangle + \frac{E}{\langle a \rangle^4} \quad (27)$$

is the mean total energy density. In this equation the dispersion $\sigma^2 = \langle a^2 \rangle - \langle a \rangle^2$ and the higher-order moments with respect to deviation of a from its mean value $\langle a \rangle$ are not taken into account. For the problems considered in the present article they can be neglected.

The mean total energy density in the state (21) equals to

$$\langle \rho \rangle = \gamma \frac{M}{\langle a \rangle^3} + \rho_{vac} + \frac{E}{\langle a \rangle^4}, \quad (28)$$

where $\gamma = 193/12$ is a numerical coefficient which appears in calculation of expectation values of the operators of the kinetic and potential parts of the energy density of the scalar field in expression (27). The mean density (28) is the sum of the energy density connected with matter (in the form of elementary quantum excitations of the vibrations of the scalar field near the equilibrium state ϕ_{vac}), the vacuum energy density and the energy density of relativistic matter.

Taking (28) into account, equation (26) can be rewritten in the form of relation for the Hubble constant $H = (1/\langle a \rangle) (d\langle a \rangle/dt)$ as a function of the cosmological redshift $z = a_0/\langle a \rangle - 1$,

$$\begin{aligned} H^2(z)/H_0^2 &= \Omega_M(1+z)^3 + \Omega_{vac} + \\ &+ \Omega_R(1+z)^4 + (1-\Omega_0)(1+z)^2, \end{aligned} \quad (29)$$

where

$$\Omega_M = \frac{\gamma M}{a_0^3 H_0^2}, \quad \Omega_{vac} = \frac{\rho_{vac}}{H_0^2}, \quad \Omega_R = \frac{E}{a_0^4 H_0^2}$$

are the components of the total energy density $\Omega_0 = \Omega_M + \Omega_{vac} + \Omega_R$ at $z = 0$, $a_0 \equiv \langle a \rangle_{z=0}$, $H_0 \equiv H(0)$. If the quantity M is assumed to be constant, then expression (29) will describe the evolution of the Universe in the model with a cosmological constant (*MCC*) represented in terms of mean values. If one establishes a direct correspondence between classical values and corresponding mean values, then such a model will be equivalent to the model with a cosmological constant of classical cosmology [9]. In this case the feedback coupling between geometry and matter given by relation (25) is not taken into consideration.

Account for (25) in (28) leads to the mean energy density ⁶

$$\langle \rho \rangle = \frac{\gamma}{\langle a \rangle^2} + \tilde{\rho}_{vac} + \tilde{\rho}_{rad}, \quad (30)$$

where we denote

$$\tilde{\rho}_{vac} = (1 - 4\gamma) \rho_{vac}, \quad \tilde{\rho}_{rad} = \left(1 - \frac{\gamma}{4}\right) \frac{E}{\langle a \rangle^4}.$$

Dependence of the Hubble constant on z in the model with the feedback coupling (*MFC*) which has no analogue in classical cosmology takes the form

$$H^2(z)/H_0^2 = \tilde{\Omega}_M(1+z)^2 + \tilde{\Omega}_{vac} + \tilde{\Omega}_R(1+z)^4, \quad (31)$$

⁶Since relation (25) connects overall characteristics of the Universe, then the energy density in the form (30) describes only its properties as a homogeneous system on very large scales. The density (30), for instance, cannot be used in calculation of fluctuations of the density near the mean value $\langle \rho \rangle$, which lead to formation of visible structures in the Universe. It is necessary to use the representation (28) in order to study such processes. (See also [10].)

where the components with tildes

$$\tilde{\Omega}_M = \frac{\gamma - 1}{a_0^2 H_0^2}, \quad \tilde{\Omega}_{vac} = (1 - 4\gamma) \Omega_{vac},$$

$$\tilde{\Omega}_R = \left(1 - \frac{\gamma}{4}\right) \Omega_R$$

satisfy the obvious equality

$$\tilde{\Omega}_M + \tilde{\Omega}_{vac} + \tilde{\Omega}_R = 1. \quad (32)$$

The total energy density at $z = 0$ equals to

$$\Omega_0 = 1 + \frac{\tilde{\Omega}_M}{\gamma - 1}. \quad (33)$$

Equation (26) with the density (30) can be integrated in an explicit form. Neglecting the contribution from relativistic matter we find

$$\begin{aligned} \langle a \rangle &= \frac{a_{in}}{2} \left(1 + \sqrt{1 + \zeta^2}\right) \left\{ e^{\sqrt{\tilde{\rho}_{vac}} \Delta t} - \right. \\ &\quad \left. - \left(\frac{\zeta}{1 + \sqrt{1 + \zeta^2}} \right)^2 e^{-\sqrt{\tilde{\rho}_{vac}} \Delta t} \right\}, \end{aligned} \quad (34)$$

where $\Delta t = t - t_{in}$ is time interval counted from some initial value t_{in} , when the scale factor is equal to $a_{in} \equiv \langle a \rangle_{t=t_{in}}$; $\zeta^2 = (\gamma - 1)/(\tilde{\rho}_{vac} a_{in}^2)$. From this it follows that

$$\frac{\ddot{\langle a \rangle}}{\langle a \rangle} = \tilde{\rho}_{vac},$$

where dots denote the second derivative with respect to time t .

According to (34) in the epoch, when $\sqrt{\tilde{\rho}_{vac}} \Delta t \ll 1$, the law of evolution of the Universe must be close to linear, $\langle a \rangle \approx \sqrt{\gamma - 1} \Delta t$. If for some redshift range $\sqrt{\tilde{\rho}_{vac}} \Delta t \sim 1$, then the Universe during the expansion on this time interval tends on average to an exponential regime, namely the expansion is realized with an acceleration.

Taking into account available current astrophysical data, it is interesting to apply the theory developed above to calculation of parameters of our Universe. Below we consider the matter-dominant era, when the contribution from $\Omega_R \sim 10^{-4}$ can be neglected.

5. Parameters of the Universe

5.1 Distance modulus of a source

If one knows $H(z)$ it is possible to calculate the luminosity distance d_L to a source with redshift z ,

$$d_L = \frac{c}{H_0} \frac{1+z}{\sqrt{\Omega_0 - 1}} \sin \left(\sqrt{\Omega_0 - 1} H_0 \int_0^z \frac{dz}{H(z)} \right) \quad (35)$$

at $\Omega_0 > 1$. In the limiting case $\Omega_0 \rightarrow 1$ relation (35) describes the luminosity distance in the spatially flat Universe. Distance modulus $\mu = m - M$ (here m and M are apparent and absolute magnitudes respectively) can be calculated with the help of the equation [38]

$$\mu = 5 \lg d_L + 25, \quad (36)$$

where d_L is taken in units of megaparsecs.

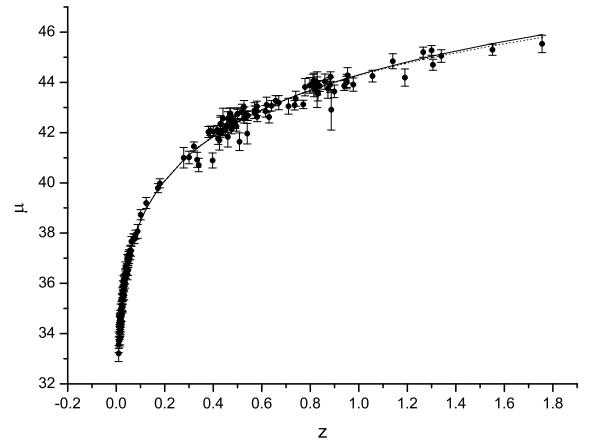


Fig. 1. Dependence of distance modulus μ (36) on redshift z . The result of best fitting (according to the χ^2 fit statistics) of quantum model (31) (with the parameter $\Omega_{vac} = -0.0075$) using SNe Ia data (dots) [5] is shown as a solid line. Model with a cosmological constant (29) is represented as a dotted line (for a flat Universe with $\Omega_{vac} = 0.71$).

In Fig. 1 we show the results of fitting (from a χ^2 statistic) of the theoretical models (MCC and MFC) for observed distance modulus μ as a function of z for 156 type Ia supernovae [5] with “high-confidence” spectroscopic and photometric record for individual source (*gold* SNe Ia in terminology of [5]). For MCC (29) (dotted curve) the best agreement between theory and observed data at $0.0104 \leq z \leq 1.755$ is achieved for $\Omega_{vac} = 0.71$ with $h = 0.65$, $\chi^2 = 178$, $\chi_{dof}^2 = 1.16$. MCC has two free parameters ($\Omega_0 = 1$ and Ω_{vac}). In this part our calculation agrees with results represented in [5]. The value $h = 0.65$ coincides with the Hubble constant which was

obtained in [3, 4, 1] according to dataset on supernovae at $z \lesssim 0.1^7$.

In the case of MFC (31) (a solid line in Fig. 1) the best agreement between theory and SNe Ia observational data is achieved at the value $\Omega_{vac} = 0.48$ which corresponds to the density parameter

$$\Omega_{vac} = -0.0075, \quad (37)$$

with $h = 0.65$, $\chi^2 = 181$, $\chi_{dof}^2 = 1.17$. MFC has only one parameter ($\tilde{\Omega}_{vac}$). Taking into account a degree of reliability of spectroscopic and photometric measurements of distant sources and their possible adjustment in the future⁸, one can conclude that both models describe distance modulus of SN Ia considered as a function of redshift z practically with the same accuracy. A susceptibility level of the χ^2 fit statistics can be judged from the following example. For $h = 0.664$ [50] we obtain $\Omega_{vac} = 0.76$ at $\chi^2 = 184$, $\chi_{dof}^2 = 1.20$ for MCC and $\tilde{\Omega}_{vac} = 0.56$ ($\Omega_{vac} = -0.0088$) at $\chi^2 = 189$, $\chi_{dof}^2 = 1.22$ for MFC. These numbers are close to mentioned above.

5.2 Energetic and geometrical scales

In the case of MCC the density parameter $\Omega_{vac} < 0$ and $|\Omega_{vac}| \ll 1^9$. This component of energy density forms the negative cosmological constant. At (37) it is equal to $\Lambda = -1.1 \times 10^{-58} \text{ cm}^{-2}$. This value is in good agreement with the available experimental data, $|\Lambda| < 10^{-56} \text{ cm}^{-2}$ [9].

The total energy density (33) equals to $\Omega_0 = 1.03$. It means that in the redshift range under consideration the Universe must look like spatially flat (to within $< 4\%$). The theoretical value of the density gets within the uncertainty limits for this value, $\Omega_0 = 1.02 \pm 0.02$ [8], obtained from the combined data of the available astronomical observations.

The scale factor in the current epoch for the obtained value of the parameter $\tilde{\Omega}_M = 0.52$ turns out to be equal to $a_0 = 24721 \text{ Mpc}$. The same value can be obtained directly from the solution (34). This number is considerably larger than the correspondent Hubble distance,

⁷Attachment of additional data from WMAP experiment [8] and HST project [4] gives $h = 0.71^{+0.04}_{-0.03}$. Since in this article we consider the problem of accelerating expansion of the Universe in the light of information on SNe Ia only, then for self-consistent calculation the value of the Hubble constant which corresponds to these data should be used.

⁸Some of the data for 172 SNe Ia represented in [4] on May, 2003 are not included in the new catalog [5] due to their low confidence with respect to one of recorded parameters (details see in [5]).

⁹Let us note that the idea of occupied levels with negative energy [51] leads to negative energy density as well [52].

$c/H_0 = 4612 \text{ Mpc}$. Such correlation between them gives the physical reason why the density Ω_0 is close to unity.

Feedback coupling relation allows to estimate the total amount of matter (the sum of masses of bodies in the Universe taken separately, without taking their gravitational interaction into account, according to equation (18)) in the present-day Universe. In dimensionless units this parameter and the scale factor are quantities of the same order of magnitude,

$$M_0 = 1.86 a_0.$$

From here we obtain $M_0 = 9 \times 10^{57} \text{ g}$ in CGS units.

5.3 Time scale

The time interval Δt counted from some instant of expansion $z = z_{in}$, taken as a reference point, to another instant fixed by observations with $z < z_{in}$ can be determined using the famous expression which follows from the definition of Hubble constant,

$$\Delta t(z) = \int_z^{z_{in}} \frac{dz}{(1+z)H(z)}.$$

Assuming $z = 0$ and $z_{in} = 1.755$, which corresponds to most distant source SN 1997ff among SNe Ia known, we obtain $H_0 \Delta t(0) = 0.74$ for MFC with the parameter (37) and $H_0 \Delta t(0) = 0.71$ for MCC with the parameters $\Omega_{vac} = 0.71$ and $\Omega_0 = 1$. This leads to practically the same time intervals, $\Delta t(0) = 11.1 \times 10^9$ years for MFC and $\Delta t(0) = 10.6 \times 10^9$ years for MCC. Supposing that expressions (29) and (31) remain valid up to singular initial state with $z = \infty$, we receive $H_0 t_0 = 1.23$ for MFC and $H_0 t_0 = 0.97$ for MCC, where $t_0 = \Delta t(0)|_{z_{in}=\infty}$ is the age of the Universe, or $t_0 = 18.5 \times 10^9$ years for MFC and $t_0 = 14.6 \times 10^9$ years for MCC. Since the parameters of both models were fitted in the finite range of z , then these values can be used for illustrative purposes only. Assuming, for example, that in the range $1.755 < z < \infty$ the Universe is described by MFC with $\Omega_{vac} = 0$ [49, 39], then in this case we have the numerical values of dimensionless time parameter and age of the Universe equal to $H_0 t_0 = 1.10$ and $t_0 = 16.5 \times 10^9$ years respectively. These numbers get within the experimental uncertainty range of correspondent parameters, $0.72 \lesssim H_0 t_0 \lesssim 1.17$ and $11 \lesssim t_0 \times 10^{-9} \text{ years}^{-1} \lesssim 17$ [9], obtained in the analysis of old stars under the assumption that stars were formed not earlier than $z = 6$.

MFC predicts the distance to SN 1997ff equal to $r_0 = c \Delta t(0) = 3396 \text{ Mpc}$. This value lays between the distances $r = 3317 \text{ Mpc}$ and $r = 5245 \text{ Mpc}$ for sources

with $z = 1$ and $z = 2$ respectively calculated in [53] for the astrophysical data in standard model with $\Omega_0 = 1$ and normalization $a_0 = c/H_0$. Using the known relation $r(t) = \chi a(t)$, where $r(t)$ is a distance to a source at the instant of time t [46], we obtain the value of the coordinate (angular distance) χ for the source SN 1997ff, $\chi = 0.137$. That is more than 20 times smaller than the maximum possible value $\chi_{max} = \pi$.

5.4 Deceleration parameter

Assuming that near $z = 0$ the deceleration parameter $q(z) = -\ddot{a}/(aH^2(z))$ can be approximated by the simple expression

$$q(z) = q(0) + z \left(\frac{dq}{dz} \right)_{z=0} \quad (38)$$

and determining the free parameters $q(0)$ and $(dq/dz)_{z=0}$ from a χ^2 statistic for SNe Ia, one can come to a conclusion [5] that the transition between the current epoch of accelerating expansion and previous phase with cosmic deceleration may take place at $z_t = 0.46 \pm 0.13$, where $q(z_t) = 0$. At the same time $q(0)$ restored by the *gold* sample of SNe Ia lays in the range from -1 to -0.5 (at the 68 % confidence level), or from -1.4 to -0.2 (at the 99 % confidence level). MCC with the parameters $\Omega_{vac} = 0.71$ and $\Omega_0 = 1$ in the approximation (38) leads to the values $q(0) = -0.57$ and $z_t = 0.46$.

For MFC (31), in approximation linear with respect to z (38) we obtain $q(0) = -0.48$ and $z_t = 0.95$. In other words both models predict an accelerating expansion of the Universe in the current epoch and a possible deceleration at $z > 1$.

From the solution (34) it follows that the inflationary expansion of the Universe, theoretically, may be realized both in the early Universe (with the large enough value of $\tilde{\rho}_{vac}$) and in later epoch. This conclusion agrees with the point of view which is widespread nowadays that the present-day Universe goes through the period of inflationary expansion again [10, 19]. (In MFC we have $\sqrt{\tilde{\rho}_{vac}} \Delta t(0) = 0.5$ in the range $0.0104 \leq z \leq 1.755$, that corresponds to observed SNe Ia.)

Let us note that the linear approximation (38) may come to an agreement with the SNe Ia data at $w < -0.5$ in equation of state $p = w\rho$, where p is pressure (for MCC $w = -1$). The more refined models which take into account a possible dependence of w on z lead to nonlinear dependence of the deceleration parameter on redshift when processing the observational data for supernovae [5].

6. Conclusion

In the present article we demonstrate that an accelerating expansion of the Universe observed nowadays for the SNe Ia data [1, 2, 5] may give the evidence in favour of presence of the small negative cosmological constant in it, $\Lambda = -1.1 \times 10^{-58} \text{ cm}^2$, and be the direct confirmation of the existence of the feedback coupling between geometry and matter on the scales that exceed significantly the size of the superclusters of galaxies anticipated by Mach's principle [48]. In quantum model of the Universe this principle is not introduced from the outside as an additional condition. It is contained in the theory by itself in the form of the condition on eigenvalues E (20). The parameters calculated in accordance with quantum mechanical principles are in good agreement with the observational data. In particular, quantum model does not contradict with an idea of the decelerating expansion of the Universe in the epoch $z > 1$. The largest possible distance between sources $r_{max} = \pi a_0 = 77663$ Mpc in the Universe described by quantum theory can be compared with the effective particle horizon 14283 Mpc calculated in [53] for the spatially flat Universe.

Exceeding the bounds of the aim of the present paper we note that quantum cosmological model allows to solve dark matter problem and give a natural explanation for presence of one more additional component in the energy density in the Universe. Here matter component of the energy density is formed as a result of a dynamic process in which elementary quantum excitations of the vibrations of primordial matter (the uniform scalar field in this article) decay into real (visible and invisible) matter mainly under the action of gravity [42, 54]. These excitations themselves are uniformly distributed in space. They practically do not interact between themselves and do not make clusters with real matter, i.e. they have properties ascribed to invisible (dark) energy [4, 6, 7], with the exception of negative pressure, perhaps. Properties of elementary quantum excitations of the vibrations of primordial matter allow to identify them with invisible energy for better reason then with invisible matter. The percentage of *matter* and *energy* components in the total energy density can be made consistent with observations on the reasonable assumptions about baryon contribution and energy released in the decay of quantum excitations [42].

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